Turbulent Heat Transfer Inside Tubes and the Analogy Among Heat, Mass, and Momentum Transfer

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The effect of fluid physical properties on the rates of convective heat transfer (or mass transfer) to Newtonian fluids flowing turbulently inside tubes has been studied extensively but never resolved for a wide range of variables. In the absence of precise experimental data the conflicting predictions of the many semitheoretical approaches developed during the last two decades have served to confuse rather than to clarify the basic question. To extend the range of the available data an experimental heat transfer study of the heretofore undefined region of high Prandtl number was performed. Fluid properties, determined experimentally, represented a variation in Prandtl number from 50 to 600.

For final correlation all the available data for heat transfer with moderate temperature differences except those on liquid metals were considered. The effect of the Prandtl number for the range of the data (N_{Pr} of 0.50 to 600) was not well represented by any of the generally accepted theories or empirical equations, although a semitheoretical correlation was deduced from the data with Reichardt's general formulation of the analogy between heat and momentum transfer. The resulting equation fits all the available data with a standard deviation of 9.4%. The applicability of the correlation to turbulent mass transfer in tubes is demonstrated for Schmidt numbers up to 3,000.

At the present time empirical design equations are normally employed for prediction of the heat transfer rates to fluids flowing turbulently inside tubes, and there is no clear choice of one in preference to another. The familiar equations of Colburn (*4*), Dittus and Boelter (*7*), Drexel and McAdams (*9*), and Sieder and Tate (*31*) are all of the form

$$N_{Nu} = c_1(N_{Re})^{c_2}(N_{Pr})^{c_3} \qquad (1)$$

but significant differences in the numerical values of c_1, c_2, and c_3 are reported.*

McAdams (*24*) noted these discrepancies and commented appropriately, "It is clear that equations of the form of the products of the powers of the three dimensionless groups, each with a constant exponent, are inadequate."

The *theory* of turbulent transport and the analogy between heat, mass, and momentum transfer have been advanced considerably during the past two decades, enabling quantitative prediction of heat or mass transfer rates in equipment of simple geometry for low and moderate values of the Prandtl group ($N_{Pr} < 20$). For viscous liquids however, for which the Prandtl number is large, the assumptions employed in the theoretical developments are subjected to a more critical test, and experimental verification is largely lacking. For extremely viscous liquids ($N_{Pr} > 100$) the literature is entirely void of any complete and accurate* experimental work, as turbulence is difficult to obtain with such fluids and they often do not conform to simple Newtonian flow behavior. Preliminary studies with viscous non-Newtonian fluids under turbulent conditions have been reported (*26*), and further work is in progress. Complete interpretation and correlation of these results in terms of fundamental arguments has not been achieved however partially owing to the undefined effect of the Prandtl number over as wide a range as may be encountered, approximately 1 to 1,000.

The ultimate goal of this investigation was to represent the rate of the heat transfer process to fluids flowing turbulently in smooth, round tubes for a broad range of fluid properties or Prandtl number, excluding from consideration only molten metals ($N_{Pr} < 0.10$) and strongly nonisothermal flow. To extend the range of the available data an experimental study of the heretofore undefined region of high Prandtl number was performed. Fluid physical properties, often only estimated by previous investigators but imperative for successful interpretation of the results, were determined experimentally. With this extension of the experimental evidence to higher values of the Prandtl number, the prior art was critically reviewed and analyzed, from the standpoint of both the theoretical mechanics of turbulent transport and the more precise and extensive data currently available. The analogy between heat, mass, and momentum transfer was examined in detail for the corresponding regions of high Schmidt and Prandtl numbers.

THEORY

The effect of the tube aspect ratio L/D has been clearly defined (*13*) and is present only in short tubes ($L/D < 20$) in which entrance effects persist over an appreciable portion of the tube length. These will be excluded from consideration here.

McAdams (*24a*) has presented the basic equations for the rates of heat (or mass) and momentum transfer and has reviewed the early theoretical attempts to obtain solutions to these equations. Experimental data on heat (or mass) transfer to fluids of high Prandtl (or Schmidt) numbers however indicate transfer rates which are much higher than those predicted by the theories. It

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*Equation (1) is rigorously applicable only for prediction of the *isothermal* heat transfer coefficient, which is defined as that limiting value which would be obtained with a zero temperature difference between the tube wall and the flowing fluid. In other words, it is that coefficient which is completely uninfluenced by complications due to the temperature dependence of the relevant physical properties of the fluid. Sieder and Tate (*31*) added an empirical correction, $(\mu_b/\mu_w)^{0.14}$, to account for radial variations in viscosity due to the radial temperature gradient. Recent data for heating aniline and *n*-butanol (*19*) in which the viscosity ratio μ_b/μ_w was varied from 2 to 14 were correlated with the correction $(\mu_b/\mu_w)^{0.10}$. McAdams (*24*) reviews the other attempts to account for deviations from this isothermal coefficient.

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*The few data available in this region are confused by the fact that high temperature differences were employed. Accordingly the resultant heat transfer coefficients are affected perhaps more strongly by complications due to the radial variation of physical properties than by the other physical processes which normally completely determine the heat transfer rates.

has been suggested (*6, 22, 30, 34*) that this failure of the theoretical solutions may be due to the assumption of an entirely laminar sublayer of fluid adjacent to the wall. Unfortunately the usual experimental methods for obtaining turbulent velocities and their fluctuations break down as one approaches the tube wall, and no direct experimental confirmation of the existence of turbulence, as defined by a finite value of the eddy diffusivity, is available for the so-called "laminar" sublayer.

Several authors (*6, 22, 30, 34*) have assumed arbitrary relationships to describe the variation of eddy diffusivity with distance from the tube wall, within this nearly laminar region close to the wall. As there is no check whatever available on such a detailed distribution, but only on its integrated effect in the over-all correlation, this practice cannot be claimed to give a necessarily correct picture of the variations in turbulence with distance from the wall. However Reichardt (*30*) deduced the general form of the correlation prior to the introduction of any such hypothetical assumptions:

$$N_{St} = \frac{f/2(\phi_m/\theta_m)}{1 + (N_{Pr} - 1)(\phi_m)\sqrt{f/2}(b)} \quad (2)$$

where

$$b = \int_0^{U/u^*} \frac{du^+}{1 + N_{Pr}E_m/\nu}$$

$$= \int_0^{U/u^*} \frac{q_{mo}}{q}\, du^+ \quad (3)$$

The development of these equations is presented in another paper (*25*), where the assumptions required are discussed in detail. Briefly, they are

1. The shear stress and heat flux are both linear functions of radial position within the tube.

2. The eddy diffusivities for heat (or mass) transfer and momentum transfer are equal.

The ratio of the mean to maximum velocities ϕ_m is nearly constant at a value of about 1/1.2 under turbulent-flow conditions (*24, 25*). The ratio of the mean to maximum temperature differences between the fluid and the tube wall θ_m is also nearly constant, provided one is dealing with Prandtl numbers greater than about unity. Therefore, by excluding the low-Prandtl-number region (that is, heat transfer to liquid metals) from consideration here, evaluation of Equation (2) is reduced to determining the value of the Reichardt b function defined by Equation (3).

Since the integrand q_{mo}/q in Equation (3) represents the fraction of the total heat transferred which is conducted by molecular motion, its value must be unity at the tube wall itself, where turbulence must be completely absent. As one moves away from the wall into the turbulent core, the ratio q_{mo}/q will decrease at a rate dependent on the Prandtl number. If one again excludes the region of very low Prandtl numbers, the numerical value of this ratio will be significant only near the wall, and the b function as a whole will be determined only by the flow conditions close to the tube wall. A detailed analysis (*12, 25*) shows that under these conditions b will be a function only of the Prandtl number, if the following criterion is met

$$(N_{Pr})(N_{Re})^2(f) > 5 \times 10^5$$

A similar analysis by Reichardt, using a less severe restriction, yielded

$$N_{Pr}N_{Re} > 2500$$

These analyses limit the theoretical development to Prandtl numbers above 0.60 and 0.25, respectively, at a Reynolds number of 10,000.

Inserting these considerations into Equation (2) one obtains as the final result

$$N_{St} = \frac{f/2}{1.20 + (N_{Pr} - 1)b\sqrt{f/2}} \quad (4)$$

where the term b is a function only of the Prandtl number and is to be determined empirically in the absence of eddy-diffusivity data close to the tube wall.

EXPERIMENTAL APPARATUS AND PROCEDURE

Heat Transfer Rates

A schematic flow diagram of the apparatus is presented in Figure 1.* The test fluid was contained in a 160-gal. stainless steel tank equipped with low- and high-speed propeller agitators and a copper coil for steam heating or water cooling. A gravity-fed Moyno *progressing cavity* type of pump with a Reeves variable-speed drive discharged the fluid into a length of ¾-in.-nominal-diameter extra-strong brass pipe, the first 10 ft. of which were insulated and served as a calming section. The following 9.90 ft. were the heated section and were insulated longitudinally from the remainder of the tube with ¼-in.-wide Bakelite rings. The wall temperature was measured by seven thermocouples distributed along the length of the heated section; the thermocouples were embedded in grooves cut deeply within the circumference of the tube. Calibrated copper-constantan thermocouple wire was used throughout. The fluid was discharged from the heated section into a series of orifice mixers equipped with a thermocouple well for measurement of the exit temperature.

The volumetric flow rate was measured with a Foxboro magnetic flowmeter and two-range dynalog recorder incorporated in the 2½-in. return line. After leaving the flowmeter the fluid passed through a double-pipe cooler, with water in the annular space, and then was returned to storage. The heater for the test section was a double-wall concentric jacket consisting of a 3-in. pipe serving as a steam jacket and a 5-in. pipe acting as a heated insulator. Slightly superheated low-pressure steam was introduced into the outer pipe and flowed into the 3-in. jacket through narrow slots cut in its surface. Cover baffles prevented condensate from dripping into the inner jacket. The inner heating jacket was provided with a small line and helical cooling coil for the removal and subcooling of condensate; the outer jacket was continuously purged from two vents, one located on the underside for condensate drainage. The heating medium was obtained by expanding a 100 lb./sq. in.-gauge line steam to about 5 lb./sq. in.-gauge, a part of the flow being diverted through a standard throttling steam calorimeter. The enthalpy change across the exchanger was determined by the calorimeter reading and the exit condensate temperature, measured prior to subcooling.

When the apparatus was running, steady-state was considered reached when both the inlet- and exit-fluid temperatures were essentially constant for a 20-min. interval. All thermocouple voltages, referred to an ice junction, were then determined with a Leeds and Northrop type-8662 portable precision potentiometer. The volumetric flow and system pressures were recorded, as well as the time required to collect between 1,000 and 2,000 cc. of condensate.

In all cases the heat transfer coefficient was based on the heat flux measured by the rate of steam condensation. Owing to the small temperature rise of the fluid through the test section, often less than 1°F., the heat flux determined by the rate of heating of the fluid was far less accurate. Even with this limitation, however, the average deviation in the two heat fluxes for the eighty experimental runs was only 9%.

The wall temperature was taken as the arithmetic average of the individual thermocouple readings. Only for the data taken with high ΔT did any one thermocouple reading differ from this average by more than 2°F., the agreement for the bulk of the data obtained with small temperature differences usually being about a few tenths of a degree.

At several times during the course of the investigation data were obtained with water (N_{Pr} = 2.2–2.5) as the test fluid. These data compared favorably with the Colburn equation, as shown in Figure 2,* over the entire range of Reynolds numbers which could be covered (from 30,000 to 420,000). This was deemed to represent excellent proof of the reliability of the experimental technique. The Colburn equation represents one of the accepted standard equations for use with fluids of low Prandtl number. Therefore its use for proving the reliability of experimental techniques may be justified on this basis (provided one uses a fluid of low Prandtl number), even though it will be shown in a later section of this paper that it and other similar equations are appreciably in error when extrapolated for use at high Prandtl numbers.

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*Figures 1 through 6 have been deposited as document 5765 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $2.50 for photoprints or $1.75 for 35-mm. microfilm.

*See footnote in column 2.

TABLE 1. SUMMARY OF FLUID PROPERTIES

Fluid	Percentage of solids	Density, lb./cu. ft.	Viscosity, centipoise	Thermal conductivity, B.t.u./(hr.)(sq. ft.)(°F.)/ft.	Specific Heat, B.t.u./(lb.)(°F.)	Prandtl number, dimensionless
Molasses: properties estimated at 180°F.						
MA	80	82.5	37	0.16	0.62	350
MB	72	81.9	15	0.17	0.64	140
MC	64	79.8	7.4	0.22	0.66	50
Corn syrup: properties estimated at 195°F.						
SA	73	83.1	60	0.14	0.59	600
SB	69	82.8	37	0.15	0.61	370
SC	65	81.7	19	0.17	0.63	180
SD	62	79.8	12	0.21	0.67	90

Fluid Physical Properties

Fluid viscosity was measured with a capillary-tube viscometer described by Metzner and Otto (*26*) and calibrated with water and a traveling microscope. The thermal conductivity of the fluids was measured by the steady state method described by Metzner, Vaughn, and Houghton (*26*). Data obtained with water in both chambers agreed with the published values (*24*) within 3%. Heat capacities were also determined in an apparatus described earlier (*26*).

The density of the fluids was measured with a hygrometer when surface properties permitted (the surface of the more concentrated fluids tended to scum and adhere strongly to the hygrometer stem); otherwise a modified pycnometric method was employed.

EXPERIMENTAL RESULTS

Fluids Studied

The fluids selected for the experimental phase of this investigation were concentrated aqueous sugar solutions, which generally exhibit the desired properties of high viscosity or high Prandtl number accompanied by perfectly Newtonian flow behavior. The Prandtl number, the variable of principal interest, could be easily changed by merely diluting the original fluid stock. The base fluids employed were a blackstrap molasses estimated as 80% sucrose and a highly refined corn syrup analyzed at 80% solids and consisting of dextrose, dextrine, maltose, and higher sugars in approximately equal proportions. The original molasses and two dilutions thereof and four dilutions of the purchased corn syrup were studied experimentally. These seven fluids, at the temperatures employed, represented a variation in the Prandtl number of 50 to 600.

Small amounts of sodium benzoate and copper nitrate were added in approximately equal proportion (total of 0.1%) to inhibit degradation of the fluid samples by fermentation. Evaporation was kept to a minimum by covering the exposed areas of the mixing tank with polyethylene film. To ensure further the stability of the fluids the data for a given fluid were obtained in the course of only a few days. Samples for the determination of physical properties were withdrawn during the runs and processed immediately thereafter. The data for the molasses-based fluids were obtained in the initial phase of the investigation, whereas the syrup-derived systems were studied with a perfected experimental technique and with smaller temperature differences; therefore they are believed to be somewhat more precise, although the agreement between the results for the two fluid groups is good.

Physical Properties

Table 1 gives a summary of the relevant properties of the seven fluids studied. The properties are estimated at approximately the bulk temperatures employed, although for the actual correlations more accurate values were estimated at the precise bulk temperature for each run from the smooth curves representing the experimental results as functions of temperature (Figures 3 to 6*). All the fluids proved to be perfectly Newtonian, with the exception of the original molasses, which exhibited slightly pseudoplastic behavior below 150°F.; at higher temperatures however it also appeared to be Newtonian.

With the exception of specific heat, fluid physical properties were measured experimentally at several temperatures. The specific heats of the molasses fluids were determined by extrapolation of data reported in *The International Critical Tables* for aqueous sucrose solutions at 68°F. as a function of composition. The specific heats of the syrups were measured experimentally at approximately 100°F. and assumed invariant with temperature. The use of these values is supported by the agreement of the heat fluxes computed independently from the rate of steam condensation and the rate of heating of the fluid, particularly for those runs in which the temperature rise of the fluid was sufficiently large to be measured precisely ($\Delta T > 2°F.$)

Fluid viscosity and density, although strongly temperature dependent, posed no particular problem, as the experimental techniques employed permitted measurement of these properties over the entire temperature range of interest. Smooth curves were drawn through the experimental results for interpolation.

Owing to limitations in the experimental procedure thermal conductivities could be measured only within the region of 90° to 150°F.,* and extrapolation of the data to the higher temperatures of interest was necessary. The data which could be obtained were of sufficient variation however to establish adequately the relatively large negative temperature coefficient of thermal conductivity exhibited by all of the fluids studied. The extrapolation to higher temperatures could therefore be performed with a fair degree of accuracy. In anticipation of possible criticism that the measured heat transfer rates are too high, compared with conventional correlations for less viscous fluids, it may be pointed out that the extrapolation was, in all cases, conservative, the curvature of the thermal conductivity-temperature curves favoring high values of the thermal conductivity. Furthermore, possible error introduced by this extrapolation is diminished in the final dimensionless correlations, as the thermal conductivity enters only to a fractional power of approximately ⅔. It is estimated that the maximum possible error in the correlation which may be attributed to thermal conductivity was always less than 10%.

Heat Transfer Rates

To avoid serious nonisothermal complications, small temperature driving forces were maintained, 14° to 19°F. for the syrup runs and 20° to 35°F. with the molasses and its dilutions. The experimental Nusselt numbers are plotted against the Reynolds numbers for the syrup and molasses runs (Figures 7 and 8 respectively). Since the bulk of the data are in the transition and early turbulent regions and the temperature potentials are small, all properties are evaluated at the average bulk temperature.

The indicated curves in the transition region, which extends to a Reynolds number certainly no greater than 10^4, are merely smooth curves through the data. For the more viscous fluids, where the experimental results terminate in the transition region, the curves have been extrapolated to $N_{Re} = 10^4$ to permit later comparison of all the data at this Reynolds number. Extrapolation of these curves was greatly facilitated by following the form established by the experimental results for the more dilute systems.

In the fully turbulent region, defined conservatively by $N_{Re} > 10^4$, all the lines shown were drawn with a slope of 0.90. This represents approximately the

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*See footnote on page 394.

*A reported value for the thermal conductivity at 150°F. actually represents a mean value over the approximate region of 110° to 190°F.; at 90°F. the range of temperature is about 65° to 115°F.

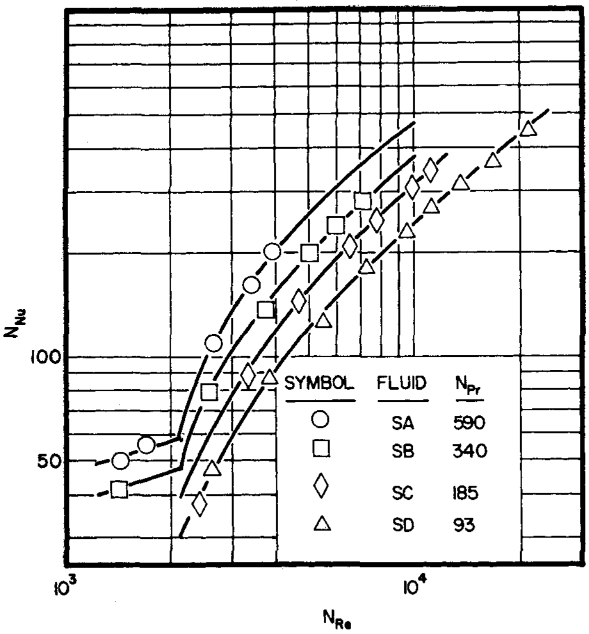

Fig. 7. Heating corn syrup (low temperature difference).

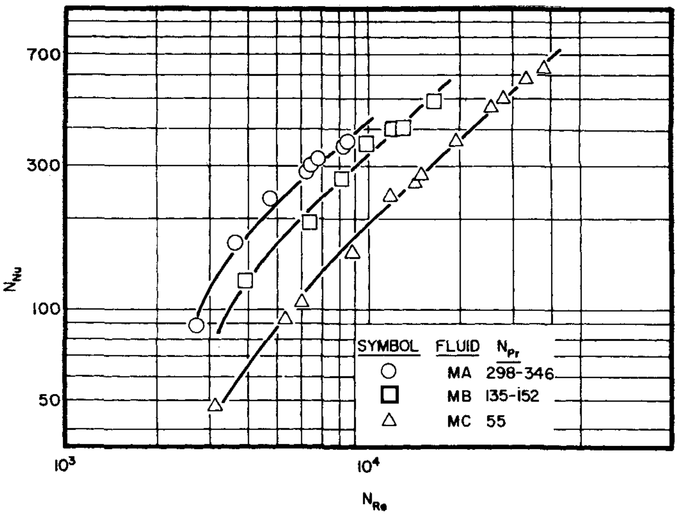

Fig. 8. Heating molasses.

behavior predicted by Equation (4): for a high Prandtl number N_{St} is proportional to $\sqrt{f/2}$, or approximately to $(N_{Re})^{-0.10}$. The Nusselt number is then proportional to $\sqrt{f/2}(N_{Re})$, or approximately to $(N_{Re})^{0.90}$. The equation predicts the conventional 0.80 Reynolds number exponent only for fluids with a Prandtl number in the vicinity of unity. The agreement of the data with this predicted effect of the Reynolds number is remarkably good. However *MC* is the only fluid for which the Reynolds number range covered is broad enough to permit a truly critical distinction. Because of unperfected technique in the earliest part of the work the average temperature and thus the Prandtl number was allowed to decrease as the flow rate and Reynolds number was increased from run to run. In this respect the earlier molasses data, fluids *MA* and *MB*, are inferior to the other results and must be described by a small range of Prandtl number for each series; whereas the data obtained with *MC* and the syrups are each adequately represented by a single value. The important conclusion to be drawn from this discussion and from Figure 7 is that the curves which were extrapolated to Reynolds numbers of 10,000 correctly portray the Nusselt group to within a few per cent. This, it will be seen later, is sufficiently accurate to define the correlation up to Prandtl numbers of 600.

Effect of the Temperature Difference

To approach the isothermal heat transfer coefficient as closely as possible heat transfer rates were measured with small temperature-driving forces, as small as could be maintained without destroying the accuracy of the measurement. The viscous fluids studied were still quite temperature sensitive, however, and even with this precaution appreciable radial variation in both viscosity and thermal conductivity were present. The Sieder-Tate correction for the viscosity gradient was approximately 1.05 for the syrup runs and ranged from 1.05 to 1.09 for the molasses data. To determine experimentally the importance of nonisothermal effects for the low ΔT data, a brief study of the effect of the temperature difference was made with the least viscous syrup, fluid *SD*.

Ideally in a study of this nature the bulk-fluid conditions are held constant and the wall temperature is changed. With this procedure the only experimental variable is the temperature difference, and its importance is readily disclosed. The apparatus employed in this work was not suited to this technique, however, and it was necessary to hold the wall temperature constant and to change the temperature difference by varying the main-stream temperature. In addition to the low ΔT data already presented, measurements of the heat transfer coefficient were made with three increased temperature differences, approximately 30°, 45°, and 55°F. The experimental Nusselt numbers are plotted vs. the Reynolds number for each of the temperature differences in Figure 9; all physical properties are evaluated at the appropriate bulk temperature.

The differences in the curves drawn in Figure 9 are due to the corresponding differences in the Prandtl number and/or the temperature potential. To isolate the effect of the ΔT it was first necessary therefore to account for the variation in the Prandtl number. For this purpose the Nusselt numbers at $N_{Re} = 10^4$ were cross-plotted against the bulk Prandtl number in Figure 10. The pertinent effect of the Prandtl number was assumed to be given by the low ΔT data for the four syrup fluids studied.

The data indicate a good correlation between the results obtained with high and low temperature differences. The high ΔT Nusselt numbers are consistently slightly smaller than the behavior indicated by the low ΔT correlation, but the difference is only 5% at the maximum temperature difference. This result may be attributed to the compensating effects of the radial variations in viscosity and thermal conductivity, as shown by the following illustrative calculation for run SD-17 (*12*):

$$N_{Re} = 6{,}290 \qquad T_b = 149.0\text{°F.}$$

$$N_{Pr} = 201 \qquad T_w = 199.5\text{°F.}$$

$$N_{Nu} = 199 \qquad T_m = 50.5\text{°F.}$$

$$\frac{\mu_b}{\mu_w} = \frac{0.0200}{0.0073} = 2.74$$

$$\frac{k_w}{k_b} = \frac{0.205}{0.240} = 0.854$$

The influence of the viscosity gradient may be estimated by the Sieder-Tate (*31*) factor $(\mu_b/\mu_w)^{0.14} = 1.15$ or by the correction recommended by Kreith and Summerfield (*19*) $(\mu_b/\mu_w)^{0.10} = 1.10$. The slope of the low ΔT correlation between the Nusselt and Prandtl numbers indicated by Figure 10 is 0.35. The heat transfer coefficient is therefore approximately proportional to $(k)^{0.65}$. When one assumes that the conductivity should be evaluated precisely at the wall, then the correction for its variation is approximately $(k_w/k_b)^{0.65} = 0.90$. The effects of thermal conductivity and viscosity are therefore of the same order of magnitude and should be approximately compensating as indicated by experiment.

In view of these considerations nonisothermal corrections were not applied to the data for final correlations, and all properties were retained at the main-

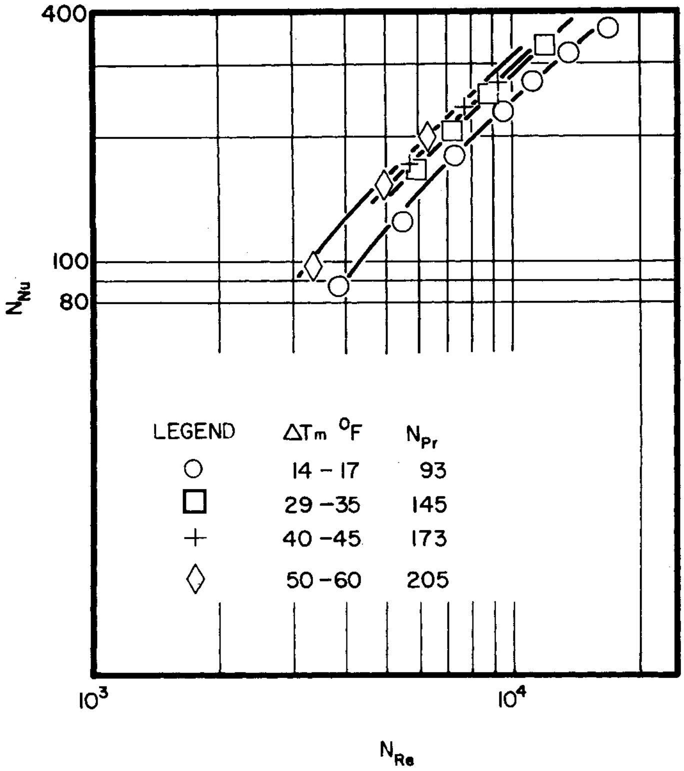

Fig. 9. Heating *SD*: (effect of the temperature difference).

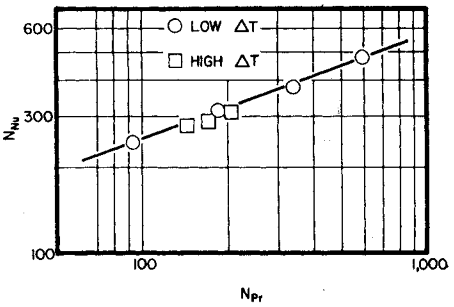

Fig. 10. Heating corn syrup, $N_{Re} = 10^4$

stream temperature. Any small effects which are present are well within the accuracy of the data and cannot be formulated quantitatively.

CORRELATION

The Effect of the Reynolds Number

The final form of dimensionless correlations for the isothermal Nusselt or Stanton number involves both the Reynolds and Prandtl groups as independent variables. The theory developed earlier completely determines the effect of the Reynolds number, however, and it remains only to demonstrate the agreement of experiment with theory on this point. The final simplified theoretical result was

$$N_{St} = \frac{f/2}{1.20 + (N_{Pr} - 1)\sqrt{f/2}(b)} \quad (4)$$

or for the Nusselt number

$$N_{Nu} = \frac{(f/2)(N_{Re}N_{Pr})}{1.20 + (N_{Pr} - 1)\sqrt{f/2}(b)} \quad (5)$$

Since f is approximately proportional to $(N_{Re})^{-0.20}$, Equation (5) predicts that for low Prandtl numbers in the neighborhood of unity the Nusselt number varies with the 0.80 power of the Reynolds number, whereas for high Prandtl number the variance is with the 0.90 power of N_{Re}. For low and moderate N_{Pr} the 0.80 exponent has long been well established by experiment. [See, for example, reference (*24*).] For high Prandtl numbers the present experimental results for heating molasses and corn syrup confirm the predicted variation. Kern (*17*) also has observed that the data of Morris and Whitman (*28*) for gas straw oils (Prandtl numbers of 35 to 100) are best correlated with the 0.90 power of the Reynolds number. Further the data of Bernardo and Eian (*1*), which cover a much wider range of Reynolds numbers then either of previously mentioned references, show a consistent increase in the value of the exponent on the Reynolds number as the Prandtl number is increased. In fact from a Prandtl number of 3 to one of 25 the value of the exponent increased by 0.10, although all the exponents were abnormally low. On the other hand the excellent mass transfer data of Eisenberg, Tobias, and Wilke (*11*), covering a thousandfold range of Reynolds numbers at extremely high Schmidt numbers, predict a variation of N_{St} (or N_{Sh}) with f rather than with $\sqrt{f}$. These investigators, however, studied rotating cylinders rather than flow inside round tubes. While a detailed recent study (*16*) revealed that the temperature fields close to a heated rotating cylinder appeared similar to those next to the hot wall of a round tube through which a fluid is flowing, the differences under consideration in high Prandtl or Schmidt number systems would quite possibly be too small to measure or might be too

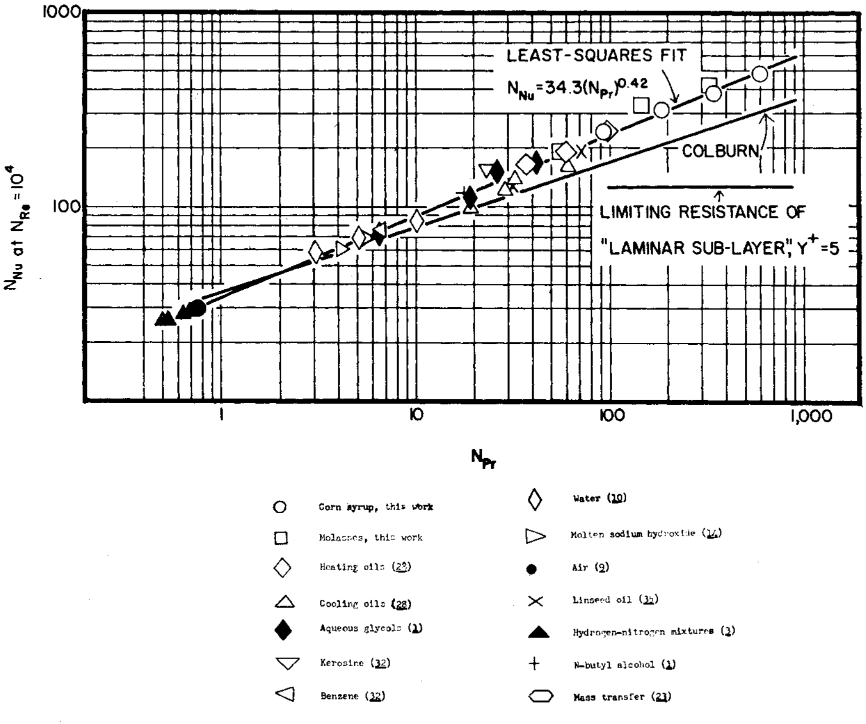

Fig. 11. Effect of the Prandtl number.

close to the wall itself. The possibility of the presence of vortices in the turbulent field around a rotating cylinder (*15*) could well be expected to lead to differences in the boundary layers as well, under conditions conductive to such vortex formations. Unfortunately one must therefore conclude that the best high Schmidt number data available are of unproved applicability to the problem at hand; while it is quite possible that *the work is* directly relevant, this would not be so under all conditions. Since by comparison the heat transfer data cover only modest ranges of Reynolds numbers, the predicted variation of heat and mass transfer rates with Reynolds number, over wide ranges, requires further study. Fortunately, highly viscous systems, which are the only ones for which the heat transter rate variations are still in doubt, probably are seldom processed in regions far removed from those of Figures 7 and 8. Therefore the remaining problem is of importance primarily in the transfer of mass rather than heat.

Analysis may now proceed to the influence of the Prandtl number by examining the experimental data at one value of the Reynolds number. For this purpose the smallest Reynolds number which definitely lies in the fully turbulent region, $N_{Re} = 10^4$, was chosen.

Data Selected

In addition to the experimental data obtained as part of this investigation *all* the available literature data for heat transfer in long tubes with low and moderate temperature differences with the single exception of those for liquid metals were assembled for final correlation. The variation in the Prandtl number for all the data considered is over more than a three-decade range, from 0.50 to 600. Especially notable among the more recent work are the data of Bernardo and Eian (*1*) for ethylene glycol–water mixtures, N_{Pr} of 6 to 40, and the study of Coghlan and Colburn (*3*) with hydrogen-nitrogen mixtures, N_{Pr} of 0.46 to 0.69.

TABLE 2. SUMMARY OF EXPERIMENTAL AND LITERATURE DATA AT $N_{Re} = 10^4$

System	Reference	N_{Pr}	N_{Nu}	Temperature effects
Heating hydrogen-nitrogen mixtures				
55.0% Hydrogen	*3*	0.46	24.8	Inestimable, believed very small
37.0% Hydrogen	*3*	0.50	26.0	
28.6% Hydrogen	*3*	0.53	26.5	
8.9% Hydrogen	*3*	0.63	28.5	
Pure nitrogen	*3*	0.69	29.0	
Air, correlation	*9*	0.74	30.0	Not considered in correlation
Heating molten	*14*	4.2	60.3	ΔT_m less than 30°F.
Sodium hydroxide	*14*	5.6	69.0	
Water	*10*	3.0	58.4	Extrapolated to zero ΔT_m
Water	*10*	5.0	70.7	
Water	*10*	10.0	84.5	
Heating benzene	*32*	5.7–7.3	77	*vc* = 1.03
Heating *n*-butanol	*1*	17.7	115	*vc* = 1.04
Heating kerosene	*32*	21–25	155	*vc* = 1.05
Heating glycol-water mixtures				
30% Glycol	*1*	6.3–6.4	72	*vc* = 1.03
70% Glycol	*1*	18.6	115	*vc* = 1.03
Pure glycol	*1*	25.3	150	*vc* = 1.04
Pure glycol	*1*	41.0	170	*vc* = 1.02
Heating gas oil	*28*	37	165	*vc* = 1.09
Cooling gas oil	*28*	18.2–19.9	110	*vc* = 0.91
Cooling gas oil	*28*	27–31	120	*vc* = 0.91
Heating straw oil	*28*	58.5	193	*vc* = 1.07
Heating straw oil	*28*	95	245	*vc* = 1.13
Cooling straw oil	*28*	32	140	*vc* = 0.90
Cooling straw oil	*28*	55–65	160	*vc* = 0.86
Heating linseed oil	*35*	31.6	127	About 60°F. ΔT_m
Heating linseed oil	*35*	70.3	189	
Heating molasses and corn syrup				
SD	This work	93	245	Less than 5%
SC	This work	185	315	Less than 5%
SB	This work	340	380	Less than 5%
SA	This work	590	480	Less than 5%
MC	This work	55	195	Less than 5%
MB	This work	135–152	330	Less than 5%
MA	This work	298–346	420	Less than 5%

For those fluids which have been studied extensively by more than one investigator the results of the investigation believed outstanding were selected: for water the data of Eagle and Ferguson (*10*) were used, inasmuch as the effect of the temperature difference was determined and the results extrapolated to zero ΔT, or the isothermal coefficient; for oils the data of Morris and Whitman (*28*); and for air the correlation of the best available data reported by Drexel and McAdams (*9*) were chosen.

The literature is markedly devoid of data for cooling, and with the exception of some data for cooling oils all of the results examined are for heating, usually with steam or electrical energy. The deviations from isothermal behavior are large for the data reported by Morris and Whitman; hence only those runs in their paper obtained with temperature differences less than 100°F. were considered. In all cases physical properties were evaluated at the bulk temperature, and nonisothermal corrections were not applied; these effects, small for most of the data considered, are within the accuracy of many of the measurements.

In the treatment of the data the experimental Nusselt numbers were plotted against the Reynolds number for a series of runs with an approximately constant Prandtl number, and smooth curves were drawn through the points. The ordinates of these curves at $N_{Re} = 10^4$ are the data considered. A summary of all the results obtained in this manner is given in Table 2, including complete references and comments pertaining to the nonisothermal conditions. For those cases in which only the viscosity varies appreciably with temperature the Sieder-Tate viscosity correction $(\mu_b/\mu_w)^{0.14}$, denoted in the table as *vc*, has been tabulated as a first estimate of this effect. This correction was not employed for correlation however and is intended only to illustrate the approximate magnitude of the effect.

Empirical Correlation

The Nusselt numbers at $N_{Re} = 10^4$ are shown in Figure 11 as a function of the Prandtl number for all of the data considered. The empirical equations of Colburn and Sieder and Tate, employing the ⅓ power of the Prandtl number, are not representative of the data at all, being only fair approximations over a small range of Prandtl numbers.* A horizontal line, representing the maximum value

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*The range from 0.50 to 20 in which the Colburn equation fits approximately is the range of variables for which Colburn claimed the equation to be useful. However common usage of this and similar empirical equations which have an exponent of approximate ⅓ on the Prandtl numbers has not been restricted accordingly. Comparison with the Dittus-Boelter equation is not made, since neither form of their equation was intended to apply to *isothermal* coefficients; hence any agreement with the data of Figure 11 would be fortuitous. It may be noted in passing, however, that the *heating* form of their equation would closely approximate the trend shown by the experimental data.

of the Nusselt number attainable at $N_{Re} = 10^4$ with the model of a purely laminar layer of thickness $y^+ = 5$, is shown for the theory of von Karman (*36*). This concept is also clearly refuted by the data.

To obtain an entirely empirical correlation a least-squares line was put through the data. Its equation is

$$N_{Nu} = 34.3(N_{Pr})^{0.42} \qquad (6)$$

To complete this conventional form of correlation by including the effect of the Reynolds number as a simple exponential function is impossible, inasmuch as the exponent probably varies with the Prandtl number. If as a first crude approximation the exponent is taken constant as 0.80, then

$$N_{Nu} = 0.022(N_{Re})^{0.80}(N_{Pr})^{0.42} \qquad (7)$$

Semitheoretical Correlation

Values of the Reichardt b function were computed by Equation (4) for each of the data points. The correlation of Drew,* Koo, and McAdams (*8*) was used to determine the friction factor

$$f = 0.00140 + 0.125(N_{Re})^{-0.32} \qquad (8)$$

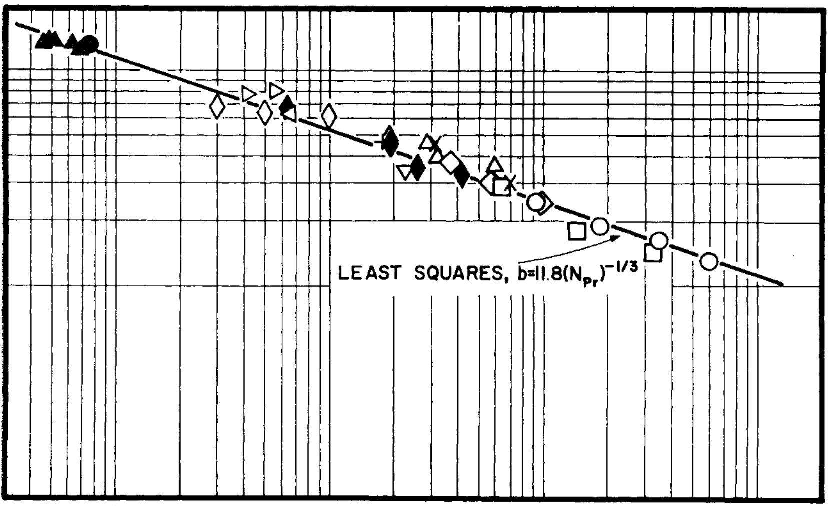

Fig. 12. Evaluation of the Reichardt *b* function. (See Figure 11 for legend.)

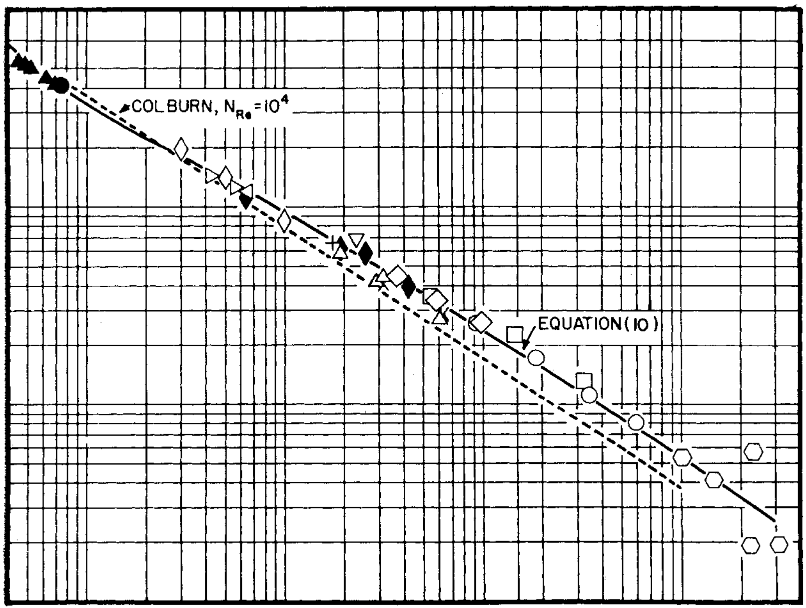

Fig. 13. Semitheoretical correlation. (See Figure 11 for legend.)

The experimental values of b are plotted vs. the Prandtl number on logarithmic coordinates in Figure 12. The data appear to be well represented by a linear relation on these coordinates, so a simple empirical exponential form $b = B(N_{Pr})^m$ was assumed. The equation of the least-squares line through the data is

$$b = 11.8(N_{Pr})^{-1/3} \qquad (9)$$

The final semitheoretical correlation for the heat transfer is then

$$N_{St} = \frac{f/2}{1.20 + 11.8\sqrt{f/2}(N_{Pr} - 1)(N_{Pr})^{-1/3}} \qquad (10)$$

Stanton numbers computed from the correlating equation are plotted vs. the Prandtl number for a Reynolds number of 10^4 in Figure 13. The Colburn equation is shown for comparison, and a design chart based on the correlation is presented in Figure 14.

The experimental results at $N_{Re} = 10^4$ are correlated with a standard deviation of 9.4%, 74% of the data falling within this limit. The maximum deviation is 22%. The correlation is believed to be a good approximation of the isothermal coefficient, inasmuch as the maximum deviations are for those data for which nonisothermal effects are strongest. Inclusion of the Sieder-Tate viscosity correction reduced the standard deviation to 6.5% and the maximum deviation to 15%. This by no means represents a general solution to the nonisothermal problem, however, and use of this correction is recommended only as an estimate of nonisothermal deviations for heat transfer with high temperature differences and fluids for which only viscosity varies appreciably with temperature.

The curves of the Stanton number vs. the Prandtl number shown in Figures 13 and 14 exhibit a point of inflection at a Prandtl number of approximately two. This representation has no theoretical significance of course; it is merely a consequence of the simple empirical form chosen for b, and the approximate treatment of the second-order effects θ_m and ϕ_m, which become increasingly important for low Prandtl numbers. Fortunately, within the range of applicability of the equation ($N_{Pr} > 0.50$) the change in slope is very slow, and the correlating equation deviates from a smooth extension of the curve at a higher Prandtl number by a maximum of only about 3%.

Application to Homogeneous Mass Transfer

Since the theoretical developments apply equally well to the turbulent diffusion of mass in a homogeneous phase, if the eddy diffusivity for mass is assumed equal to the eddy diffusivity for heat and momentum, one may write as the correlating equation for turbulent mass transfer in tubes

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*Over a small range of Reynolds numbers the Blasius equation (*2*) could also have been used.

$$\left(\frac{k_L}{V}\right) = \frac{f/2}{1.20 + 11.8\sqrt{f/2}(N_{Sc} - 1)(N_{Sc}^{*})^{-1/3}} \tag{11}$$

The excellent agreement between heat and mass transfer data for low and moderate values of the Schmidt and Prandtl numbers (0.50 to 20) is well known (*6, 33*). There is little point therefore in an extensive review of this data; rather, attention was directed exclusively to the undefined region of very high Schmidt numbers for which two important investigations have been reported.

Linton and Sherwood (*23*) measured rates of solution of various organic salts coated on the inside wall of a tube. The Schmidt number was varied from 1,000 to 3,000. Their data were considered here in the same manner as the heat transfer results, and the smooth values of (k_L/V) at $N_{Re} = 10^4$ are plotted against the Schmidt number in Figure 13. The data scatter appreciably but generally tend to support the proposed correlation. This agreement is especially important, as it constitutes support of the correlation beyond the range of the heat transfer data upon which it was based and permits an approximate extension of the range of applicability of the results to Prandtl numbers as great as 3,000.

A more recent study of a similar problem was performed by Lin and coworkers at the University of Washington (*21*). This work is of particular interest, as the results are quoted by Deissler (*6*) and Lin, Moulton, and Putnam (*22*) in support of their analogies. Mass transfer coefficients were measured for the diffusion of ions in a diffusion-controlled electrolytic system. The solutions flowed through an annular space ($D_2/D_1 = 2$) between two concentric electrodes. The data for the laminar region are reported as agreeing with the "Leveque (*20*) approximation,"

$$\frac{k_L D_e}{D_L} = 1.62\left(\frac{G D_e^2}{\rho D_L L}\right)^{1/3} \tag{12}$$

while the turbulent data for Schmidt numbers of 300 to 3,000 fit the Colburn equation

$$\frac{k_L}{V}(N_{Sc})^{2/3} = 0.023\left(\frac{D_e V \rho}{\mu}\right)^{-0.20} \tag{13}$$

The results in the turbulent region agree with the correlation presented here as to the effect of the Schmidt number for high values of this group, but they are about 30% lower than those predicted by Equation (11). In view of the apparent disagreement between these widely quoted mass transfer data and the proposed correlation, the results of the investigation of Lin et al. (*21*) will be discussed in further detail.

Generally one should not assume that transport phenomena in annuli may be represented by the same equations for flow in tubes by merely replacing the tube diameter with the equivalent diameter D_e ($= D_2 - D_1$). From a purely dimensional viewpoint the pi theorem requires that the consideration of an additional variable in the dimensional analysis of a problem without including an additional dimension be offset by the inclusion of an additional dimensionless group—for flow in an annulus (D_2/D_1). The theoretical equation for laminar friction factors (*18*) as well as the empirical equations for turbulent friction (*5, 37*) and for turbulent heat transfer (*27*) all contain functions of this group.

Although Leveque's result for flow inside tubes does not apply, the basic principle of the Leveque approximation under laminar flow conditions should

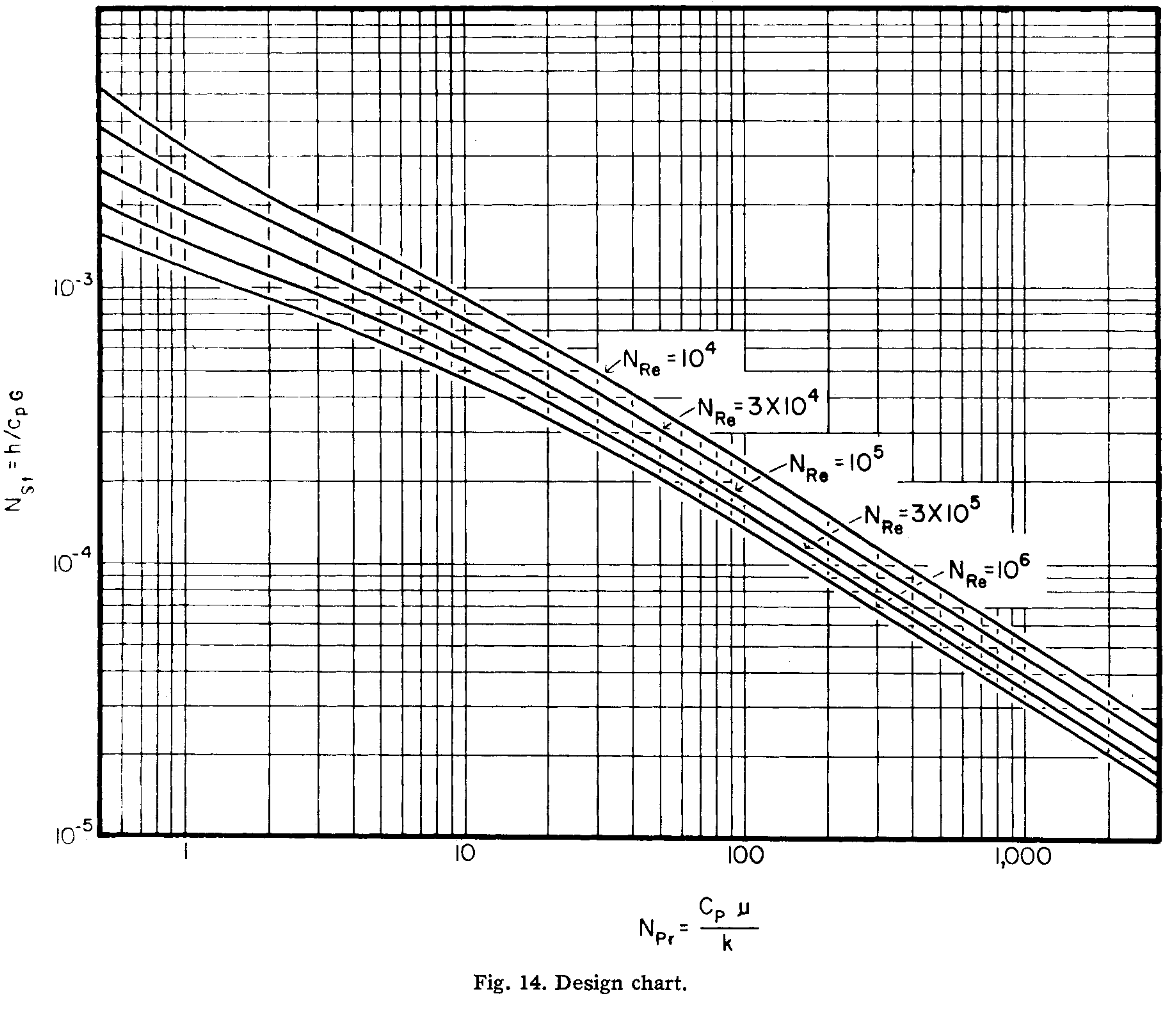

Fig. 14. Design chart.

apply to annular flow for the conditions of high mass flow rates through short tubes. The difference in the final result arises from the difference in the laminar-velocity distributions for the two distinct geometrical configurations. The appropriate derivation* has been performed (*12*), and the final result is

$$\frac{k_L D_1}{D_L} = 1.75\left(\frac{w}{\rho D_L L}\right)^{1/3} [F(D_1/D_2)]^{1/3} \tag{14}$$

where $F(D_1/D_2)$ is given by

$$F(D_1/D_2) = \frac{(D_1/D_2)^2}{1-(D_1/D_2)^2} \cdot \left[\frac{0.50-\left\{\frac{(D_1/D_2)^2}{1-(D_1/D_2)^2}\right\}\ln\frac{D_2}{D_1}}{\left(\frac{1+(D_1/D_2)^2}{1-(D_1/D_2)^2}\right)\left(\ln\frac{D_2}{D_1}\right)-1}\right] \tag{15}$$

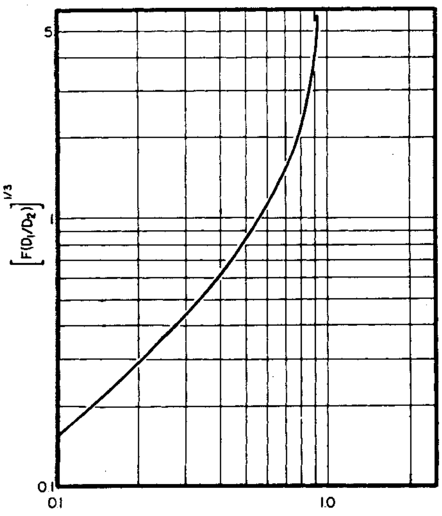

Fig. 15. ***F*(D_1/D_2)-Leveque approximation for annuli.**

To aid the use of these theoretical results they are shown graphically in Figure 15. For $D_1/D_2 = 0.50$ the theory predicts

$$k_L D_1/D_L = 1.46\left(\frac{w}{\rho D_L L}\right)^{1/3} \tag{16}$$

whereas the data in question are correlated by

$$k_L D_1/D_L = 1.24\left(\frac{w}{\rho D_L L}\right)^{1/3} \tag{17}$$

The experimental results of Lin et al. are thus approximately 15% lower than the proper theoretical relation in the laminar region, and they are 22% lower than the best empirical relation (*27*) in the turbulent region. Clearly the use of these data cannot be recommended.

In the absence of further data for mass transfer the agreement of the data of Linton and Sherwood for high Schmidt number as well as the bulk of the data for low and moderate Schmidt number are accepted as confirmation of the proposed correlation. The analogies of Deissler and of Lin, Moulton, and Putnam which fit the disputed annular data of Lin et al. are believed to predict coefficients which are too low in the high Prandtl or Schmidt range. The difference between their coefficients and those predicted by the present work is about 30% in the range of Prandtl numbers between 1,000 and 3,000 when based on the present coefficient or 45% when based on values given by the earlier correlations.

CONCLUSIONS AND RECOMMENDED DESIGN EQUATIONS

1. The generally accepted empirical correlations and the semitheoretical analogies between heat and momentum transfer derived with experimental data for low and moderate Prandtl number are not representative of heat transfer in high Prandtl-number systems. In particular the model of a perfectly laminar wall layer is clearly refuted. The modified analogies of Deissler (*6*), Lin, Moulton, and Putnam (*22*), and Rannie (*34*), although significant improvements, predict coefficients which are still appreciably too small in the high Prandtl or Schmidt number regions.

2. The best empirical equation for the turbulent heat transfer of the simple exponential form at $N_{Re} = 10^4$ is

$$N_{Nu} = 34.4(N_{Pr})^{0.42}$$

This relation was found to correlate heat transfer data over a range of Prandtl numbers from 0.50 to 600 and is extended by approximate mass transfer data to Prandtl numbers of 3,000. If, as a first approximation, the conventional 0.80 exponent of the Reynolds number is assumed for the entire range of Prandtl numbers, then

$$N_{Nu} = 0.022(N_{Re})^{0.80}(N_{Pr})^{0.42}$$

However for fluids with high Prandtl numbers this empirical equation becomes conservative, as the exponent on the Reynolds number appears to increase toward 0.90. A design chart, based on the precise relationship, is given in Figure 14.

3. The theoretical solution indicated by the analogy between heat and momentum transfer, with the exception of the region of extremely low Prandtl number, is of the form

$$N_{St} = \frac{f/2}{1.20 + \sqrt{f/2}(N_{Pr} - 1)(b)}$$

where the Reichardt *b* function is determined exclusively by the Prandtl number if $(N_{Pr})(N_{Re})^2(f) > 5 \times 10^5$.

4. The Reichardt *b* function is given empirically for a Prandtl number range of 0.50 to 600 by $b = 11.8\ (N_{Pr})^{-1/3}$. The final semitheoretical correlation

$$N_{St} = \frac{f/2}{1.20 + 11.8\sqrt{f/2}(N_{Pr} - 1)(N_{Pr})^{-1/3}}$$

represents all the available data for heat transfer with low and moderate temperature differences with a standard deviation of 9.4%. Nonisothermal deviations may arise from strong radial variations in both viscosity and thermal conductivity. If only viscosity varies appreciably with temperature, these may be approximately estimated with the correction $(\mu_b/\mu_w)^{0.14}$ or $(\mu_b/\mu_w)^{0.10}$

5. The corresponding result for liquid-phase mass transfer in tubes

$$\left(\frac{k_L}{V}\right) = \frac{f/2}{1.20 + 11.8\sqrt{f/2}(N_{Sc} - 1)(N_{Sc})^{-1/3}}$$

is supported by the bulk of the available data for low and moderate Schmidt numbers and the data of Linton and Sherwood (*23*) for Schmidt numbers of 1,000 to 3,000. The widely quoted annular flow data of Lin et al. (*21*) are not directly applicable to pipe flow, and furthermore they consistently deviate by approximately 15 and 22% from the proper theoretical relation in the laminar region and the best empirical equation in the turbulent region, respectively.

RECOMMENDATIONS FOR FURTHER WORK

The final semitheoretical correlation presented in this work is believed to be a satisfactory representation of the limiting or isothermal coefficient of heat transfer. Nonisothermal deviations may be only crudely estimated with the present state of the art, however, and future studies of strongly nonisothermal flow should prove enlightening. Extremely precise experimental techniques would be required, and of first interest would be a fluid for which only viscosity varies with temperature. Studies with gases for which the specific heat and density

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*A similar approach with some empirical corrections for natural convection was used by Rao (*29*) to correlate laminar heat transfer data for water in an annulus.

change relatively rapidly with temperature would also be of fundamental interest.

The problem of non-Newtonian heat transfer appears amenable to a theoretical approach similar to that used herein. If a suitable expression for the molecular shear such as the empirical power-law model is incorporated in the momentum transfer equation, manipulation of the equations for the fluxes of *heat and momentum* to indicate the form of the final solution for the heat transfer should be possible. Empirical representation of the unknown functions or, if possible, theoretical computation from basic turbulent flow measurements would then complete the solution.

ACKNOWLEDGMENT

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NOTATION

Any consistent set of units may be employed. Those listed are merely illustrative.

b = Reichardt b function defined by Equation (3) and given experimentally by 11.8 $(N_{Pr})^{-1/3}$, dimensionless

c_1, c_2, c_3 = empirical constants, dimensionless

C_p = specific heat, B.t.u./(lb.-mass)(°F.)

d = differential operator

D = tube diameter, ft.

D_1 = outside diameter of inner tube in concentric annulus, ft.

D_2 = inside diameter of outer tube in concentric annulus, ft.

D_e = equivalent diameter, ft.

D_L = liquid molecular diffusion coefficient, sq. ft./hr.

E_m = eddy diffusivity for momentum, sq. ft./hr.

f = fanning friction factor, dimensionless

F = function of (D_1/D_2) in equation for laminar heat or mass transfer in annuli

g_c = conversion factor 32.17 $(\text{lb.}_m)(\text{ft.})/(\text{lb.}_F)(\text{sec.}^2)$

G = mass velocity, lb.-mass/(hr.)(sq. ft.)

h = film coefficient of heat transfer, B.t.u./(hr.)(sq. ft.)(°F.)

k = thermal conductivity, B.t.u./(hr.)(ft.)(°F.)

k_L = liquid phase mass transfer coefficient, moles/(sec.)(sq. ft.)(moles)/cu. ft.

L = tube length, ft.

q = heat flux, B.t.u./(hr.)(sq. ft.)

T = temperature, °F.

ΔT = temperature difference or tempeature rise, °F.

u = time average point velocity in x direction, ft./sec.

u^* = friction velocity, $\sqrt{g_c\tau/\rho}$, ft./sec.

u^+ = generalized velocity parameter, u/u^*, dimensionless

U = maximum linear velocity at center line, ft./sec.

V = mean linear flow velocity, ft./sec.

w = mass flow, lb.-mass/hr.

Greek Letters

θ = dimensionless temperature difference, $T - T_o/T_c - T_o$

θ_m = ratio of mean to maximum temperature differences

μ = viscosity, lb.-mass/ft. sec. or lb.-mass/(ft.)(hr.)

ν = kinematic viscosity, sq. ft./hr.

ρ = fluid density, lb.mass/cu. ft.

τ = shear stress, lb.-force/sq. ft.

ϕ = dimensionless velocity, u/U

ϕ_m = ratio of mean to maximum velocities, V/U

Subscripts

b = bulk average

c = center line

mo = molecular

w = wall

Dimensionless Numbers

N_{Nu} = (hD/k) Nusselt number

N_{Pr} = $(C_p\mu/k)$ Prandtl number

N_{Re} = (DG/μ) Reynolds number

N_{Sc} = $(\mu/\rho D_L)$ Schmidt number

N_{Sh} = (k_L/V) Sherwood number

N_{St} = (h/C_pG) Stanton number

LITERATURE CITED

1. Bernardo, Everett, and C. S. Eian, *Nat. Advisory Comm. Aeronaut.*, ARR No. E5F07 (1945).
2. Blasius, H., *Forsch. Arb. Ing. Wes.*, **131** (1913).
3. Coglan, Charles, M.Ch.E. thesis, Univ. of Delaware, Newark (1940).
4. Colburn, A. P., *Trans. Am. Inst. Chem. Engr.*, **29**, 174 (1933).
5. Davis, E. S., *Trans. Am. Soc. Mech. Engrs.*, **65**, 755 (1943).
6. Deissler, R. G., *Nat. Advisory Comm. Aeronaut.*, Report 1210 (1955).
7. Dittus, P. W., and L. M. K. Boelter, *Univ. of Calif.* Engr. Publ. **2**, 443 (1930). (Through reference 24).
8. Drew, T. B., E. C. Koo, and W. H. McAdams, *Trans. Am. Inst. Chem. Engrs.*, **28**, 56 (1933).
9. Drexel, R. E., and W. H. McAdams, *Nat. Advisory Comm. Aeronaut.*, ARR. No. 4F28, also Wartime report 108 (1945).
10. Eagle, Albert, and R. M. Ferguson, *Proc. Roy. Soc. London*, **A127**, 540 (1930).
11. Eisenberg, Morris, C. W. Tobias, and C. R. Wilke, *Chem. Eng. Progr. Symposium Series* No. 16, **51**, 1 (1955).
12. Friend, W. L., M.Ch.E. thesis, Univ. of Delaware, Newark (1957).
13. Hartnett, J. P., *Trans. Am. Sec. Mech. Engrs.*, **77**, 1211 (1955); Deissler, R. G., *ibid.*, p. 1221; and Kays, W. M., *ibid.*, p. 1265.
14. Hoffman, H. W., paper presented at 1953 Heat Transfer and Fluid Mechanics Institute, Univ. of S. Calif. (1953).
15. Kay, Joseph and E. C. Elgar, *Trans. Am. Soc. Mech. Engrs.*, **80**, 753 (1958).
16. Kays, W. M. and I. S. Bjorklund, *ibid.*, **80**, 70 (1958).
17. Kern, D. Q., "Process Heat Transfer," p. 46, McGraw-Hill Book Company, Inc., New York (1950).
18. Knudsen, J. G., and D. L. Katz, "Fluid Dynamics and Heat Transfer," Eng. Res. Inst., Univ. of Mich. (1954).
19. Kreith, Frank, and Martin Summerfield, *Trans. Am. Soc. Mech. Engrs.*, **72**, 869 (1950).
20. Leveque, M. A., *Ann. mines*, **13**, 201 (1928).
21. Lin, C. S., E. B. Denton, H. S. Gaskill, and G. L. Putnam, *Ind. Eng. Chem.* **43**, 2136 (1951).
22. Lin, C. S., R. W. Moulton, and G. L. Putnam, *ibid.*, **45**, 636 (1953).
23. Linton, W. H., and T. K. Sherwood, *Chem. Eng. Progr.*, **46**, 258 (1950).
24. McAdams, W. H., "Heat Transmission," 3 ed., McGraw-Hill Book Company, Inc., New York (1954).

24a. *Ibid.*, p. 208.

25. Metzner, A. B. and W. L. Friend, *Can. J. Chem. Eng.* (In Press).
26. Metzner, A. B., in "Advances in Chemical Engineering," vol. I, Adademic Press, New York (1956). Also Metzner, A. B., R. D. Vaughn, and G. L. Houghton, *A.I.Ch.E. Journal* **3**, 92 (1957); and Metzner, A. B., and R. E. Otto, *A.I.Ch.E. Journal* **3**, 3 (1957).
27. Monrad, C. C., and J. F. Pelton, *Trans. Am. Inst. Chem. Engrs.*, **38**, 593 (1942).
28. Morris, F. H., and W. G. Whitman, *Ind. Eng. Chem.*, **20**, 234 (1928).
29. Rao, C. V., Ph.D. thesis, Univ. of Delaware, Newark (1949).
30. Reichardt, H., "Fundamentals of Turbulent Heat Transfer," trans. from *Arch. ges. Warmtech.*, No. 6/7 (1951), *Nat. Advisory Comm. Aeronaut.*, TM-1408 (1957), and N-41947 (1956).
31. Sieder, E. N., and G. E. Tate, *Ind. Eng. Chem.*, **28**, 1429 (1936).
32. Sherwood, T. K., and J. M. Petrie, *ibid.*, **24**, 736 (1932).
33. Sherwood, T. K., and R. L. Pigford, "Absorption and Extraction," McGraw-Hill Book Company, Inc., New York (1952).
34. Summerfield, Martin, Heat Transfer Symposium, Eng. Res. Inst., Univ. of Mich., 151 (1953) [through reference (*6*)].
35. Ullock, D. S., and W. L. Badger, *Ind. Eng. Chem.*, **29**, 905 (1937).
36. von Karman, Theodore, *Trans. Am. Soc. Mech. Engrs.*, **61**, 705 (1939).
37. Walker, J. E., G. A. Whan, and R. R. Rothfus, *A.I.Ch.E. Journal*, **3**, 484 (1957).

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